Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

0296910415

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

[Turn over

1 The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

2 Find the exact solution of the equation

$$\tfrac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}\left(\tfrac{1}{2}\sqrt{3}\right). \qquad [2]$$

3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P. [2]

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(b) Find the equation of the curve. [4]

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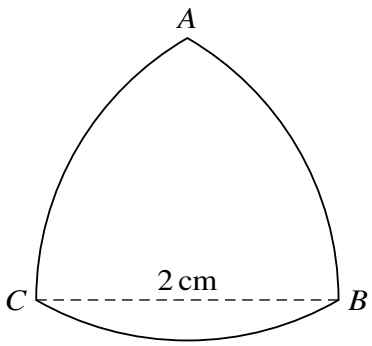

The diagram shows the shape of a coin. The three arcs AB, BC and CA are parts of circles with centres C, A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

(a) Find the perimeter of the coin. [2]

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(b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

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5 The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

6 The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

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The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

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(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

7 **(a)** Verify the identity $(2x - 1)(4x^2 + 2x - 1) \equiv 8x^3 - 4x + 1$. [1]

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(b) Prove the identity $\dfrac{\tan^2\theta + 1}{\tan^2\theta - 1} \equiv \dfrac{1}{1 - 2\cos^2\theta}$. [3]

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(c) Using the results of **(a)** and **(b)**, solve the equation

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 4\cos\theta,$$

for $0° \leqslant \theta \leqslant 180°$. [5]

8 Functions f and g are defined by

$$\mathrm{f}(x) = (x + a)^2 - a \text{ for } x \leqslant -a,$$
$$\mathrm{g}(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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(b) **(i)** State the domain of the function f^{-1}. [1]

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(ii) State the range of the function f^{-1}. [1]

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(c) Given that $a = \frac{7}{2}$, solve the equation $\mathrm{gf}(x) = 0$. [3]

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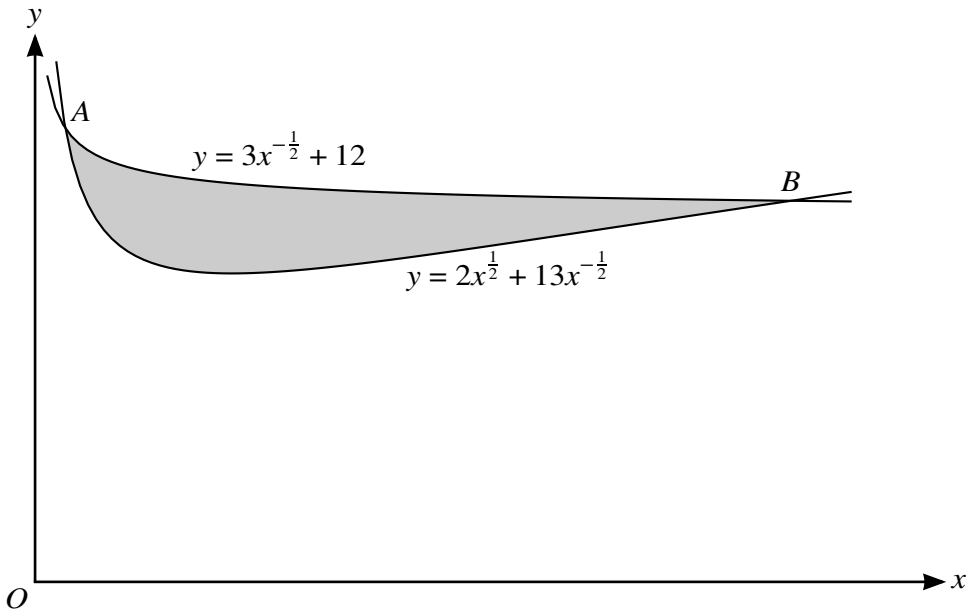

The diagram shows curves with equations $y = 2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}$ and $y = 3x^{-\frac{1}{2}} + 12$. The curves intersect at points A and B.

(a) Find the coordinates of A and B. [4]

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(b) Hence find the area of the shaded region. [5]

10 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (4x - 3)^{\frac{5}{3}} - \frac{20}{3}x$.

(a) Find the x-coordinates of the stationary points of the curve and determine their nature. [6]

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(b) State the set of values for which the function f is increasing. [1]

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11 The coordinates of points A, B and C are $(6, 4)$, $(p, 7)$ and $(14, 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC.

(a) Given that $p < 10$, find the value of p. [4]

A circle passes through the points A, B and C.

(b) Find the equation of the circle. [3]

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(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d, e and f are integers. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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