Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4535261400

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 06_9709_32/RP

[Turn over

1 Solve the inequality $|2x - 1| < 3|x + 1|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - \mathrm{i}| \leqslant 1$ and $\arg(z - 1) \leqslant \frac{3}{4}\pi$. [4]

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

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It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

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4 Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) \mathrm{d}x$. [5]

5 The complex number u is given by $u = 10 - 4\sqrt{6}\mathrm{i}$.

Find the two square roots of u, giving your answers in the form $a + \mathrm{i}b$, where a and b are real and exact. [5]

6 **(a)** Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan\theta$. [3]

(b) Hence show that $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta)\,\mathrm{d}\theta = \tfrac{1}{2}\ln 2$. [4]

7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, \mathrm{e})$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [7]

8 The equation of a curve is $y = \mathrm{e}^{-5x} \tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x-coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

9 Let $f(x) = \dfrac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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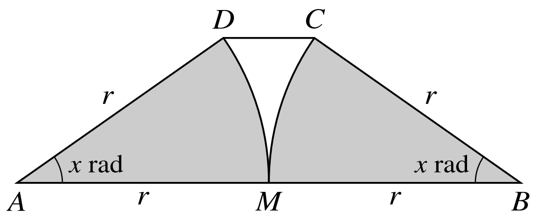

The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M, the midpoint of AB.

(a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x)\sin x$. [3]

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(b) Verify by calculation that x lies between 0.5 and 0.7. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(2 - \frac{x_n}{0.9 \sin x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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(d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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11 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

(a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

The midpoint of AB is M. The point P on the line through O and M is such that $PA : OA = \sqrt{7} : 1$.

(b) Find the possible position vectors of P. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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