# Cambridge Assessment International Education

## Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

**MATHEMATICS** **9709/22**

Paper 2 Pure Mathematics 2 **February/March 2022**

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

**[Turn over**

**1** Solve the equation $|5x - 2| = |4x + 9|$. [3]

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**2** A curve has equation $y = 7 + 4\ln(2x + 5)$.

Find the equation of the tangent to the curve at the point $(-2, 7)$, giving your answer in the form $y = mx + c$. [5]

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**3** The variables $x$ and $y$ satisfy the equation $y = 3^{2a}a^x$, where $a$ is a constant. The graph of $\ln y$ against $x$ is a straight line with gradient 0.239.

**(a)** Find the value of $a$ correct to 3 significant figures. [3]

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**(b)** Hence find the value of $x$ when $y = 36$. Give your answer correct to 3 significant figures. [2]

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**4 (a)** Show that $\sin 2\theta \cot \theta - \cos 2\theta \equiv 1$. [3]

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**(b)** Hence find the exact value of $\sin \frac{1}{6}\pi \cot \frac{1}{12}\pi$. [2]

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**(c)** Find the smallest positive value of $\theta$ (in radians) satisfying the equation

$$\sin 2\theta \cot \theta - 3\cos 2\theta = 1.$$ [2]

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**5** **(a)** Given that $y = \tan^2 x$, show that $\dfrac{dy}{dx} = 2\tan x + 2\tan^3 x$. [2]

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**(b)** Find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\tan x + \tan^2 x + \tan^3 x)\,dx$. [6]

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**6** The polynomial p($x$) is defined by

$$p(x) = 4x^3 + 16x^2 + 9x - 15.$$

**(a)** Find the quotient when p($x$) is divided by $(2x + 3)$, and show that the remainder is $-6$. [3]

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**(b)** Find $\displaystyle\int \frac{p(x)}{2x + 3}\, dx$. [2]

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**(c)** Factorise p($x$) + 6 completely and hence solve the equation

$$p(\operatorname{cosec} 2\theta) + 6 = 0$$

for $0° < \theta < 135°$. [5]

**7** A curve has equation $e^{2x}y - e^y = 100$.

**(a)** Show that $\dfrac{dy}{dx} = \dfrac{2e^{2x}y}{e^y - e^{2x}}$. [3]

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**(b)** Show that the curve has no stationary points. [2]

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It is required to find the $x$-coordinate of $P$, the point on the curve at which the tangent is parallel to the $y$-axis.

**(c)** Show that the $x$-coordinate of $P$ satisfies the equation

$$x = \ln 10 - \tfrac{1}{2}\ln(2x - 1).$$

[4]

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**(d)** Use an iterative formula, based on the equation in part **(c)**, to find the $x$-coordinate of $P$ correct to 3 significant figures. Use an initial value of 2 and give the result of each iteration to 5 significant figures.

[3]

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**Additional Page**

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